

# Geometry - Definitions, Postulates, Properties & Theorems

## Chapter 1 & 2 – Basics of Geometry & Reasoning and Proof

### Definitions

1. Congruent Segments (p19)
2. Congruent Angles (p26)
3. Midpoint (p35)
4. Angle Bisector (p36)
5. Vertical Angles (p44)
6. Complementary Angles (p46)
7. Supplementary Angles (p46)
8. Perpendicular Lines (p79)

### Postulates

1. **Ruler Postulate:** The points on a line can be matched one to one with the real numbers. The real numbers that correspond to a point is the coordinate of the point. The distance between points  $A$  and  $B$ , written as  $AB$ , is the absolute value of the difference between the coordinates  $A$  and  $B$ .
2. **Segment Addition Postulate:** If  $B$  is between  $A$  and  $C$ , then  $AB + BC = AC$ , then  $B$  is between the coordinates of  $A$  and  $C$ .
3. **Protractor Postulate:** Consider a point  $A$  on one side of  $\overline{OB}$ . The rays of the form  $\overline{OA}$  can be matched one to one with the real numbers from 0 to 180. The measure of  $\angle AOB$  is equal to the absolute value of the difference between the real numbers for  $\overline{OA}$  and  $\overline{OB}$ .
4. **Angle Addition Postulate:** If  $P$  is in the interior of  $\angle RST$ , then  $m\angle RSP + m\angle PST = m\angle RST$ .

#### Point, Line, and Plane Postulates

5. Through any two points there exists exactly one line.
6. A line contains at least two points.
7. If two lines intersect, then their intersection is exactly one point.
8. Through any three noncollinear points there exists exactly one plane.
9. A plane contains at least three noncollinear points.
10. If two points lie in a plane, then the line containing them lies in the plane.
11. If two planes intersect, then their intersection is a line.
12. **Linear Pair Postulate:** If two angles form a linear pair, then they are supplementary.

### Properties

#### Algebraic Properties of Equality

Let  $a$ ,  $b$ , and  $c$  be real numbers.

1. **Addition Property:** If  $a = b$ , then  $a + c = b + c$
2. **Subtraction Property:** If  $a = b$ , then  $a - c = b - c$
3. **Multiplication Property:** If  $a = b$ , then  $ac = bc$
4. **Division Property:** If  $a = b$  and  $c \neq 0$ , then  $\frac{a}{c} = \frac{b}{c}$
5. **Reflexive Property:** For any real number  $a$ ,  $a = a$
6. **Symmetric Property:** If  $a = b$ , then  $b = a$
7. **Transitive Property:** If  $a = b$  and  $b = c$ , then  $a = c$
8. **Substitution Property:** If  $a = b$ , then  $a$  can be substituted for  $b$  in any equation or expression.
9. **Distributive Property:**  $a(b + c) = ab + ac$

#### Properties of Equality – Segment Length

- a. **Reflexive:** For any segment  $AB$ ,  $AB = AB$ .
- b. **Symmetric:** If  $AB = CD$ , then  $CD = AB$ .
- c. **Transitive:** If  $AB = CD$  and  $CD = EF$ , then  $AB = EF$ .

#### Properties of Equality – Angle Measure

- a. **Reflexive:** For any angle  $A$ ,  $m\angle A = m\angle A$ .
- b. **Symmetric:** If  $m\angle A = m\angle B$ , then  $m\angle B = m\angle A$ .
- c. **Transitive:** If  $m\angle A = m\angle B$  and  $m\angle B = m\angle C$ , then  $m\angle A = m\angle C$ .

### Theorems

#### 2.1 Properties of Segment Congruence Theorem

- a. **Reflexive:** For any segment  $AB$ ,  $\overline{AB} \cong \overline{AB}$ .
- b. **Symmetric:** If  $\overline{AB} \cong \overline{CD}$ , then  $\overline{CD} \cong \overline{AB}$ .
- c. **Transitive:** If  $\overline{AB} \cong \overline{CD}$  and  $\overline{CD} \cong \overline{EF}$ , then  $\overline{AB} \cong \overline{EF}$ .

#### 2.2 Properties of Angle Congruence Theorem

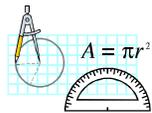
- a. **Reflexive:** For any angle  $A$ ,  $m\angle A \cong m\angle A$ .
- b. **Symmetric:** If  $m\angle A \cong m\angle B$ , then  $m\angle B \cong m\angle A$ .
- c. **Transitive:** If  $m\angle A \cong m\angle B$  and  $m\angle B \cong m\angle C$ , then  $m\angle A \cong m\angle C$ .

**2.3 Right Angle Congruence Theorem:** All right angles are congruent.

**2.4 Congruent Supplements Theorem:** If two angles are supplementary to the same angle (or to congruent angles) then the two angles are congruent.

**2.5 Congruent Complements Theorem:** If two angles are complementary to the same angle (or to congruent angles) then the two angles are congruent.

**2.6 Vertical Angles Theorem:** Vertical angles are congruent.



# Geometry - Definitions, Postulates, Properties & Theorems

## Chapter 3 – Perpendicular and Parallel Lines

### Definitions

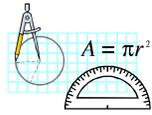
1. **Parallel Lines** (p129) – two lines that are coplanar and do not intersect. (The symbol for “is parallel to” is  $\parallel$ )
2. **Skew Lines** (p129) – two lines that do not intersect and are not coplanar.
3. **Transversal** (p131) – a line that intersects two or more coplanar lines at different points.

### Postulates

13. **Parallel Postulate:** If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.
14. **Perpendicular Postulate:** If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.
15. **Corresponding Angles Postulate:** If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.
16. **Corresponding Angles Converse:** If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.
17. **Slopes of Parallel Lines:** In a coordinate plane, two non-vertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.
18. **Slopes of Perpendicular Lines:** In a coordinate plane, two non-vertical lines are perpendicular if and only if the product of their slopes is  $-1$ . Vertical and horizontal lines are perpendicular.

### Theorems

- 3.1 If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.
- 3.2 If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.
- 3.3 If two lines are perpendicular, then they intersect to form four right angles.
- 3.4 **Alternate Interior Angles:** If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.
- 3.5 **Consecutive Interior Angles:** If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.
- 3.6 **Alternate Exterior Angles:** If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.
- 3.7 **Perpendicular Transversal:** If a transversal is perpendicular to one of two perpendicular lines, then it is perpendicular to the other.
- 3.8 **Alternate Interior Angles Converse:** If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.
- 3.9 **Consecutive Interior Angles Converse:** If two lines are cut by a transversal so that consecutive interior angles are supplementary, then the lines are parallel.
- 3.10 **Alternate Exterior Angles Converse:** If two lines are cut by a transversal so that alternate exterior angles are congruent, then the lines are parallel.
- 3.11 If two lines are parallel to the same line, then they are parallel to each other.
- 3.12 In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.



# Geometry - Definitions, Postulates, Properties & Theorems

## Chapter 4 & 5 – Congruent Triangles & Properties of Triangles

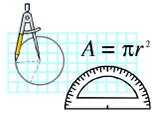
### Postulates

19. **Side-Side-Side (SSS) Congruence Postulate:** If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.
20. **Side-Angle-Side (SAS) Congruence Postulate:** If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.
21. **Angle-Side-Angle (ASA) Congruence Postulate:** If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

### Theorems

- 4.1 **Triangle Sum Theorem:** The sum of the measures of the interior angles of a triangle is  $180^\circ$ .  
**Corollary:** The acute angles of a right triangle are complementary.
- 4.2 **Exterior Angle Theorem:** The measure of an exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles.
- 4.3 **Third Angles Theorem:** If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.
- 4.4 **Properties of Congruent Triangles**
  - 4.4.1 *Reflexive:* Every triangle is congruent to itself.
  - 4.4.2 *Symmetric:* If  $\triangle ABC \cong \triangle DEF$ , then  $\triangle DEF \cong \triangle ABC$ .
  - 4.4.3 *Transitive:* If  $\triangle ABC \cong \triangle DEF$  and  $\triangle DEF \cong \triangle JKL$ , then  $\triangle ABC \cong \triangle JKL$ .
- 4.5 **Angle-Angle-Side (AAS) Congruence Theorem:** If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.
- 4.6 **Base Angles Theorem:** If two sides of a triangle are congruent, then the angles opposite them are congruent.  
**Corollary:** If a triangle is equilateral, then it is equiangular.
- 4.7 **Base Angles Converse:** If two angles of a triangle are congruent, then the sides opposite them are congruent.  
**Corollary:** If a triangle is equiangular, then it is equilateral.
- 4.8 **Hypotenuse-Leg (HL) Congruence Theorem:** If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and leg of a second right triangle, then the two triangles are congruent.

- 5.1 **Perpendicular Bisector Theorem:** If a point is on a perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.
- 5.2 **Perpendicular Bisector Converse:** If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.
- 5.3 **Angle Bisector Theorem:** If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.
- 5.4 **Angle Bisector Converse:** If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.
- 5.5 **Concurrency of Perpendicular Bisectors of a Triangle:** The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.
- 5.6 **Concurrency of Angle Bisectors of a Triangle:** The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.
- 5.7 **Concurrency of Medians of a Triangle:** The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.
- 5.8 **Concurrency of Altitudes of a Triangle:** The lines containing the altitudes of a triangle are concurrent.
- 5.9 **Midsegment Theorem:** The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long.
- 5.10 If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.
- 5.11 If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.
- 5.12 **Exterior Angle Inequality:** The measure of an exterior angle of a triangle is greater than the measure of either of the two non-adjacent interior angles.
- 5.13 **Triangle Inequality:** The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- 5.14 **Hinge Theorem:** If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.
- 5.15 **Hinge Theorem Converse:** If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.



# Geometry - Definitions, Postulates, Properties & Theorems

## Chapter 6 – Quadrilaterals

### Definitions

1. Parallelogram (p330) – a quadrilateral with both pairs of opposite sides parallel.
2. Rhombus (p347) – a parallelogram with four congruent sides.
3. Rectangle (p347) – a parallelogram with four congruent angles.
4. Square (p347) – a parallelogram with four congruent sides and four congruent angles.
5. Trapezoid (p356) – a quadrilateral with exactly one pair of parallel sides.
6. Kite (p358) – a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

### Postulates

22. **Area of a Square Postulate:** The area of a square is the square of the length of its side.  $A = s^2$
23. **Area Congruence Postulate:** If two polygons are congruent, then they have the same area.
24. **Area Addition Postulate:** The area of a region is the sum of the areas of its non-overlapping parts.

### Theorems

- 6.1 **Interior Angles of a Quadrilateral:** The sum of the measures of the interior angles of a quadrilateral is  $360^\circ$ .
- 6.2 If a quadrilateral is a parallelogram, then its opposite sides are congruent.
- 6.3 If a quadrilateral is a parallelogram, then its opposite angles are congruent.
- 6.4 If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.
- 6.5 If a quadrilateral is a parallelogram, then its diagonals bisect each other.
- 6.6 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- 6.7 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- 6.8 If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.
- 6.9 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- 6.10 If one pair of opposite sides of a quadrilateral is congruent and parallel, then the quadrilateral is a parallelogram.
- 6.11 A parallelogram is a rhombus if and only if its diagonals are perpendicular.
- 6.12 A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.
- 6.13 A parallelogram is a rectangle if and only if its diagonals are congruent.
- 6.14 If a trapezoid is isosceles, then each pair of base angles is congruent.
- 6.15 If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.
- 6.16 A trapezoid is isosceles if and only if its diagonals are congruent.
- 6.17 **Midsegment Theorem for Trapezoids:** The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the length of the bases.
- 6.18 If a quadrilateral is a kite, then its diagonals are perpendicular.
- 6.19 If a quadrilateral is a kite, then exactly one pair of opposite angles is congruent.
- 6.20 **Area of a Rectangle:** The area of a rectangle is the product of its base and height.  $A = bh$
- 6.21 **Area of a Parallelogram:** The area of a parallelogram is the product of a base and its corresponding height.  $A = bh$
- 6.22 **Area of a Triangle:** The area of a triangle is one half the product of a base and its corresponding height.  $A = \frac{1}{2}bh$
- 6.23 **Area of a Trapezoid:** The area of a trapezoid is one half the product of the height and the sum of the bases.  

$$A = \frac{1}{2}h(b_1 + b_2)$$
- 6.24 **Area of a Kite:** The area of a kite is one half the product of the lengths of its diagonals.  $A = \frac{1}{2}d_1d_2$
- 6.25 **Area of a Rhombus:** The area of a rhombus is one half the product of the lengths of its diagonals.  $A = \frac{1}{2}d_1d_2$